Unit Root Assignment

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1 Processing

1.1 Inputting Data

```
data <- read_table('data.txt', col_names = c("gnpbg", "gnpr", "pop"))
```

```
##
## -- Column specification --------------------------------------------------------
## cols(
##   gnpbg = col_double(),
##   gnpr = col_double(),
##   pop = col_double()
## )
```

1.2 Creating variables

```
processed <- data %>% mutate(
  lgnprpc = log(gnpr/pop),
  lgnpbgpc = log(gnpbg/pop),
  year = seq(1869, 1993)
)
nandp <- processed %>% filter(year >= 1909 & year <= 1970)
```

2 Analysis

2.1 Plotting

In Figure 1, I plotted both lgnprpc It does seem to have a relatively linear trend. While there are upsets in the trend line certainly, it overall follows a pretty linear pattern.

```
ggplot(processed, aes(y = lgnprpc, x = year)) +
  geom_line() +
  labs(title='lgnprpc') +
  theme(plot.title = element_text(hjust=0.5)) +
  scale_x_continuous(breaks = pretty(processed$year, n = 10))
```

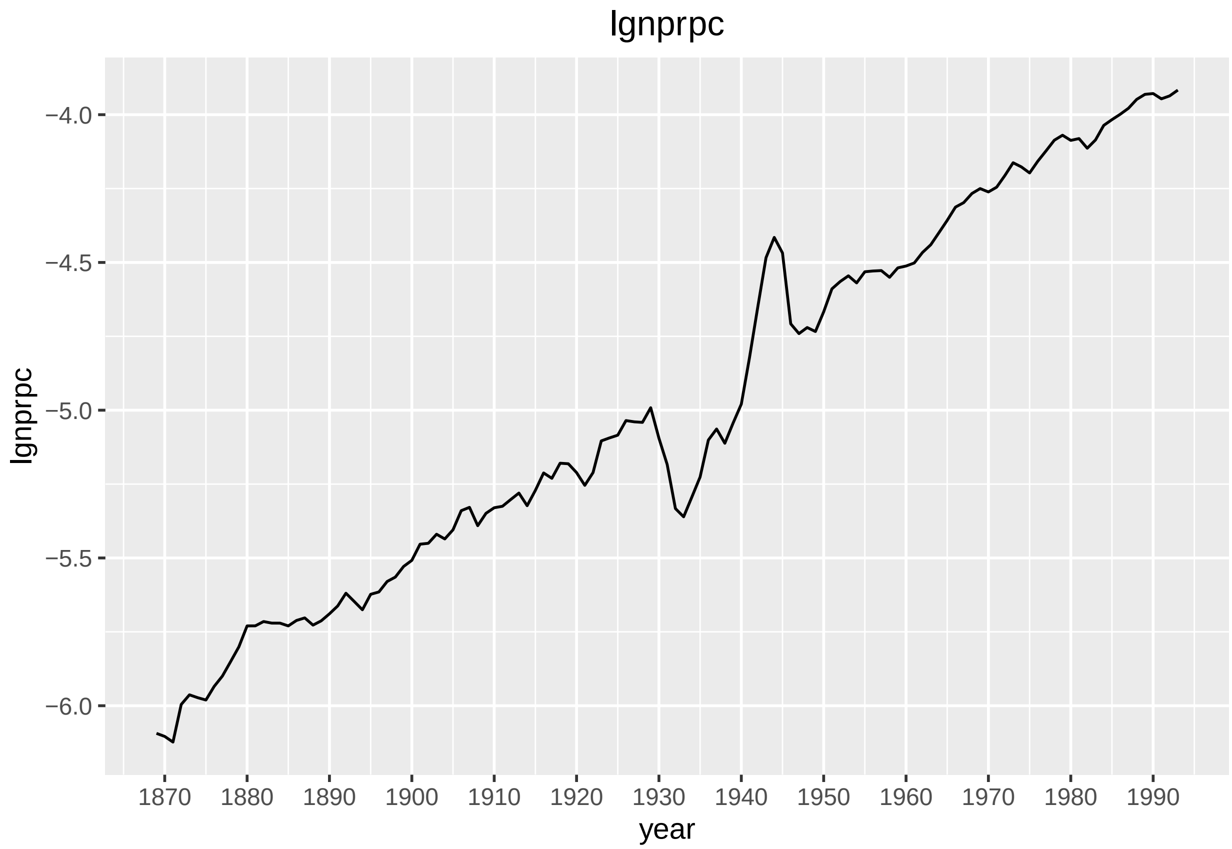

Figure 1: Graph of lgnprpc over time

2.2 Diebold

2.2.1 ADF Tests

```
adf.test(ts(processed$lgnprpc), k = 1)
```

2.2.1.1 lgnprpc

```
## Warning in adf.test(ts(processed$lgnprpc), k = 1): p-value smaller than printed
## p-value
```

```
##
##  Augmented Dickey-Fuller Test
##
## data:  ts(processed$lgnprpc)
## Dickey-Fuller = -4.6348, Lag order = 1, p-value = 0.01
## alternative hypothesis: stationary
```

```
adf.test(ts(processed$lgnprpc), k = 2)
```

```
## Warning in adf.test(ts(processed$lgnprpc), k = 2): p-value smaller than printed
## p-value
```

```
##
##  Augmented Dickey-Fuller Test
##
## data:  ts(processed$lgnprpc)
## Dickey-Fuller = -4.2707, Lag order = 2, p-value = 0.01
## alternative hypothesis: stationary
```

```
adf.test(ts(processed$lgnprpc), k = 3)
```

```
##
##  Augmented Dickey-Fuller Test
##
## data:  ts(processed$lgnprpc)
## Dickey-Fuller = -3.8882, Lag order = 3, p-value = 0.017
## alternative hypothesis: stationary
```

```
adf.test(ts(processed$lgnprpc))
```

```
##
##  Augmented Dickey-Fuller Test
##
## data:  ts(processed$lgnprpc)
## Dickey-Fuller = -3.2516, Lag order = 4, p-value = 0.08253
## alternative hypothesis: stationary
```

```
adf.test(ts(processed$lgnpbgpc), k = 1)
```

2.2.1.2 lgnpbgpc

```
## Warning in adf.test(ts(processed$lgnpbgpc), k = 1): p-value smaller than printed
## p-value
```

```
##
##  Augmented Dickey-Fuller Test
##
## data:  ts(processed$lgnpbgpc)
## Dickey-Fuller = -4.3167, Lag order = 1, p-value = 0.01
## alternative hypothesis: stationary
```

```
adf.test(ts(processed$lgnpbgpc), k = 2)
```

```
## Warning in adf.test(ts(processed$lgnpbgpc), k = 2): p-value smaller than printed
## p-value
```

```
##
##  Augmented Dickey-Fuller Test
##
## data:  ts(processed$lgnpbgpc)
## Dickey-Fuller = -4.5, Lag order = 2, p-value = 0.01
## alternative hypothesis: stationary
```

```
adf.test(ts(processed$lgnpbgpc), k = 3)
```

```
##
##  Augmented Dickey-Fuller Test
##
## data:  ts(processed$lgnpbgpc)
## Dickey-Fuller = -3.829, Lag order = 3, p-value = 0.01987
## alternative hypothesis: stationary
```

```
adf.test(ts(processed$lgnpbgpc))
```

```
##
##  Augmented Dickey-Fuller Test
##
## data:  ts(processed$lgnpbgpc)
## Dickey-Fuller = -3.4051, Lag order = 4, p-value = 0.05694
## alternative hypothesis: stationary
```

The ADF tests display mixed results. At lower difference levels, it clearly rejects the null hypothesis, which points towards it being stationary. At the difference level automatically picked by R's internal algorithm, however, it can only reject the null with a p-value of .083, which is not enough to reject the null. At 1,2 and 3 lags, however, you can easily reject the null so overall I would side towards it being stationary for sure. If using the lgnpbgpc dataset, this same thing holds true, but the fourth lag has p=.057, which is even closer to significant. I just decided to run them all instead of choosing specifically one.

2.3 N&P Sample

2.3.1 ADF Tests

```
adf.test(ts(nandp$lgnprpc), k = 1)
```

2.3.1.1 lgnprpc

```
##
##  Augmented Dickey-Fuller Test
##
## data:  ts(nandp$lgnprpc)
## Dickey-Fuller = -3.6236, Lag order = 1, p-value = 0.03862
## alternative hypothesis: stationary
```

```r
adf.test(ts(nandp$lgnprpc), k = 2)
```

```
##
##  Augmented Dickey-Fuller Test
##
## data:  ts(nandp$lgnprpc)
## Dickey-Fuller = -3.2367, Lag order = 2, p-value = 0.08997
## alternative hypothesis: stationary
```

```r
adf.test(ts(nandp$lgnprpc))
```

```
##
##  Augmented Dickey-Fuller Test
##
## data:  ts(nandp$lgnprpc)
## Dickey-Fuller = -2.9415, Lag order = 3, p-value = 0.1938
## alternative hypothesis: stationary
```

```r
adf.test(ts(nandp$lgnpbgpc), k = 1)
```

2.3.1.2 lgnpbgpc

```
##
##  Augmented Dickey-Fuller Test
##
## data:  ts(nandp$lgnpbgpc)
## Dickey-Fuller = -3.4906, Lag order = 1, p-value = 0.04986
## alternative hypothesis: stationary
```

```r
adf.test(ts(nandp$lgnpbgpc), k = 2)
```

```
##
##  Augmented Dickey-Fuller Test
##
## data:  ts(nandp$lgnpbgpc)
## Dickey-Fuller = -3.2254, Lag order = 2, p-value = 0.09177
## alternative hypothesis: stationary
```

```r
adf.test(ts(nandp$lgnpbgpc))
```

```
##
##  Augmented Dickey-Fuller Test
##
## data:  ts(nandp$lgnpbgpc)
## Dickey-Fuller = -2.9356, Lag order = 3, p-value = 0.1962
## alternative hypothesis: stationary
```

When restricting to the range that N&P used, the result is even less clear. With only one round of differencing, I find that it is stationary. When doing any more than one round, I find that it is not, and it gets quite insignificant. So I would lean on concluding that I cannot reject the null hypothesis of it having a unit root. The smaller sample size makes a difference for sure, as it is just not large enough to get statistically significant results. Also, during that period there is an upset in the graph, as the great depression and world war 2 happened, which makes it far less consistently linear during that time.

2.4 ADF-GLS Tests

2.4.1 N&P Dataset

```
summary(ur.ers(ts(nandp$lgnprpc),model='trend'))
```

```
##
## ###############################################
## # Elliot, Rothenberg and Stock Unit Root Test #
## ###############################################
##
## Test of type DF-GLS
## detrending of series with intercept and trend
##
##
## Call:
## lm(formula = dfgls.form, data = data.dfgls)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.157217 -0.026923 -0.002262  0.026686  0.110971
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## yd.lag       -0.16041    0.07152  -2.243 0.029198 *
## yd.diff.lag1  0.53887    0.12964   4.157 0.000121 ***
## yd.diff.lag2 -0.02031    0.14882  -0.136 0.892000
## yd.diff.lag3  0.05059    0.14657   0.345 0.731370
## yd.diff.lag4 -0.15786    0.13726  -1.150 0.255364
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.05605 on 52 degrees of freedom
## Multiple R-squared:  0.3846, Adjusted R-squared:  0.3255
## F-statistic: 6.501 on 5 and 52 DF,  p-value: 9.169e-05
##
##
## Value of test-statistic is: -2.2428
##
## Critical values of DF-GLS are:
##                  1pct  5pct 10pct
## critical values -3.58 -3.03 -2.74
```

```
summary(ur.ers(ts(nandp$lgnpbgpc),model='trend'))
```

```
##
## ###############################################
## # Elliot, Rothenberg and Stock Unit Root Test #
## ###############################################
##
## Test of type DF-GLS
## detrending of series with intercept and trend
##
##
## Call:
## lm(formula = dfgls.form, data = data.dfgls)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.158853 -0.026740 -0.001158  0.031182  0.109338
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## yd.lag       -0.169464   0.075855  -2.234 0.029804 *
```

```
## yd.diff.lag1  0.483782   0.131873   3.669 0.000575 ***
## yd.diff.lag2  0.005540   0.146222   0.038 0.969922
## yd.diff.lag3  0.009498   0.144373   0.066 0.947802
## yd.diff.lag4 -0.126177   0.137373  -0.919 0.362597
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.06169 on 52 degrees of freedom
## Multiple R-squared:  0.3313, Adjusted R-squared:  0.267
## F-statistic: 5.152 on 5 and 52 DF,  p-value: 0.0006512
##
##
## Value of test-statistic is: -2.2341
##
## Critical values of DF-GLS are:
##                  1pct  5pct 10pct
## critical values -3.58 -3.03 -2.74
```

Even with the ADF-GLS tests, the critical values just aren't significant enough to be able to say with the lower sample size and skewed data from depression. Below I will test it with the full dataset.

2.4.2 Full Dataset

```
summary(ur.ers(ts(processed$lgnprpc),model='trend'))
```

```
##
## ###############################################
## # Elliot, Rothenberg and Stock Unit Root Test #
## ###############################################
##
## Test of type DF-GLS
## detrending of series with intercept and trend
##
##
## Call:
## lm(formula = dfgls.form, data = data.dfgls)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.16757 -0.02082  0.00237  0.02528  0.10423
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## yd.lag       -0.15421    0.04750  -3.247  0.00153 **
## yd.diff.lag1  0.49333    0.08842   5.580 1.62e-07 ***
## yd.diff.lag2 -0.03325    0.09623  -0.345  0.73036
## yd.diff.lag3  0.03111    0.09385   0.331  0.74088
## yd.diff.lag4 -0.10488    0.08964  -1.170  0.24443
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.04295 on 115 degrees of freedom
## Multiple R-squared:   0.31,  Adjusted R-squared:   0.28
## F-statistic: 10.33 on 5 and 115 DF,  p-value: 3.35e-08
##
##
## Value of test-statistic is: -3.2468
##
## Critical values of DF-GLS are:
##                  1pct  5pct 10pct
## critical values -3.46 -2.93 -2.64
```

```r
summary(ur.ers(ts(processed$lgnpbgpc),model='trend'))
```

```
## 
## ############################################### 
## # Elliot, Rothenberg and Stock Unit Root Test # 
## ############################################### 
## 
## Test of type DF-GLS 
## detrending of series with intercept and trend 
## 
## 
## Call:
## lm(formula = dfgls.form, data = data.dfgls)
## 
## Residuals:
##       Min        1Q    Median        3Q       Max 
## -0.171522 -0.024867  0.003911  0.030585  0.119931 
## 
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)    
## yd.lag       -0.18826    0.05479  -3.436 0.000823 ***
## yd.diff.lag1  0.34682    0.09076   3.821 0.000216 ***
## yd.diff.lag2  0.13057    0.09341   1.398 0.164846    
## yd.diff.lag3 -0.06644    0.09417  -0.706 0.481894    
## yd.diff.lag4 -0.04927    0.09310  -0.529 0.597672    
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## 
## Residual standard error: 0.05029 on 115 degrees of freedom
## Multiple R-squared:  0.2297, Adjusted R-squared:  0.1962 
## F-statistic: 6.859 on 5 and 115 DF,  p-value: 1.233e-05
## 
## 
## Value of test-statistic is: -3.4359 
## 
## Critical values of DF-GLS are:
##                  1pct  5pct 10pct
## critical values -3.46 -2.93 -2.64
```

With the full dataset, it becomes very significant! N&P's dataset was simply very limited.

3 Does GNP have a unit root?

Overall, I would heavily lean towards saying no. With the full dataset, and especially with the ADF-GLS tests, you can reject the null hypothesis of difference stationary, so no unit root. It simply fits better with the available data. This was not possible on more limited datasets, however. I think the main reason for this is a mix of sample size, and more importantly, the great depression. With a difference stationary dataset, you would not have expected a bounceback to the trend line in GNP. With a trend stationary dataset, you would expect a recovery back to the trendline. With the great depression and recovery afterwards returning it (somewhat) to trend, it is harder to deny. While their dataset did include some of this, most of the data was during the recovery and rebound itself, and there wasn't a lot of baseline data to represent the longer term trend. The full dataset combines the great depression with a long term trendline both before and afterwards continuing for a long time.